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HEAT TRANSFER AND PRESSURE DROP CHARACTERISTICS OF MULTI TUBES IN TUBE HELICALLY COILED HEAT EXCHANGER

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Abstract

The present article reports on experimental investigation of heat transfer and pressure drop characteristics of multi tubes in tube helically coiled heat exchanger. The study aims to investigate the effects of heat exchanger geometric parameters and fluid flow parameters; namely, number of inner tubes, annulus hydraulic diameter, Reynolds numbers and input heat flux, on performance of the heat exchanger. Different coils with different numbers of inner tubes, namely, 1, 3, 4 and 5 tubes, were tested. Results showed that: (a) coil with three inner tubes has bitter heat transfer characteristics as compared with other coils, (b) coils with 3 and 4 inner tubes, respectively, have higher values of $\bar{h}A_h$ (the parameter used to measure performance and compactness of compact heat exchanger) as compared to other coils, (c) for all coils and at the tested ranges of Reynolds number and heat flux, input heat flux has no effect on heat transfer coefficients, and (d) pressure drop increases with increasing

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both of Reynolds number and number of inner tubes. Correlations of average Nusselt number were deduced from experimental data in terms of Reynolds number, Prandtl number and coil hydraulic diameter. Correlations prediction was compared with experimental data and the comparison was fair enough.

Nomenclature

A_c	Annulus cross sectional area of test section, m^2
C_p	Specific heat of cooling water, J/kg.K
De	Dean number, dimensionless
D_h	Annulus hydraulic diameter, m
D_i	Annulus inner diameter, m
D_o	Annulus outer diameter, m
d_o	Inner rod heater outside diameter, m
$\bar{h}$	Average convection heat transfer coefficient, $\mathrm{W/m^2.K}$
$\bar{h}A_h$	Compactness factor, W/K
I	Applied current to rod heater, A
k_w	Water thermal conductivity, W/m.K
L	Heater rod length, m
$\dot{m}_w$	Cooling water mass flow rate, kg/s
$\overline{Nu}$	Average Nusselt number, dimensionless
n	Number of inner rod heaters
Pr	Prandtl number, dimensionless
Q	Heat transfer rate, W
R	Curvature ratio, dimensionless

Re	Reynolds number, dimensionless
T_s	Average surface temperature of heater rods, °C
T_{si}	Average surface temperature of heater rod at inlet of test section, °C
T_{so}	Average surface temperature of heater rod at exit of test section, °C
T_w	Mean temperature of cooling water between inlet and outlet, °C
T_{wi}	Cooling water temperature at inlet of test section, °C
T_{wo}	Cooling water temperature at exit of test section, °C
V	Applied voltage to rod heater, V
$\dot{V}$	Water volumetric flow rate, $\mathrm{m^3/s}$
ΔP	Pressure drop, Pa
P_w	Density of water, $\mathrm{kg/m^3}$
μ_w	Dynamic viscosity of water

1. Introduction

Helically coiled heat exchangers can be found in a wide range of engineering applications including food processing, nuclear reactors, compact heat exchangers, heat recovery systems, chemical processing, refrigeration and air conditioning systems and medical equipment. Helically coiled heat exchanger is very alluring for various engineering processes because its accommodation of large heat transfer area in a small space with high heat transfer coefficients. Tube curvature in helically coiled heat exchangers induces secondary flow patterns which enhance heat transfer between tube wall and flowing fluid. To enhance the main two advantages of helical coiled heat exchangers: high heat transfer surface area per occupied size and high heat transfer rate due to the induced secondary flow, multi tubes in tube helical heat exchangers as shown in Figure 1 is proposed. Multi tubes in tube helical heat exchangers, characterized as liquid to liquid compact heat exchangers, can accommodate large heat transfer surface area in a smaller size. While multi tubes in tube helically coiled heat exchangers are presented in market and used in a lot of engineering applications, fluid flow and heat transfer characteristics of such type of heat exchangers are not published yet. To the author's knowledge, no data are available in the wide literature regarding the performance of such types of heat exchangers.

Figure 1. A schematic diagram of multi tubes in tube helical heat exchanger.

The present study aims to experimentally investigate fluid flow and heat transfer characteristics of multi tubes in tube helical heat exchanger. Parameters that can be used to measure performance of this type of heat exchanger are also presented, investigated and estimated. Effects of some geometric parameters of the heat exchangers, such as the number of inner tubes inside the helical outer tube and the effects of fluid flow parameters on the performance of the heat exchanger are also investigated.

Although no data are available in literature for multi tubes in tube helical heat exchangers, extensive work has been published on flow and heat transfer characteristics in helical/curved pipes and in annulus of double pipe helical heat exchangers.

Flow in curved pipes

Secondary flow profile in helical tube was firstly investigated and described by Dean [1]. He showed the occurrence of a swirling flow pattern separated by a horizontal centre line. Dean presented the Dean number (*De*), ratio of inertial to viscous forces, to characterize secondary flow profile. Dravid et al. [2] reported that the characteristics of outward directed flow originate at the centre point and depend on Dean number. Akiyama and Cheng [3] numerically studied the condition of a steady fully developed laminar forced convection in uniformly heated curved pipes for a range of Dean numbers up to about 200. Their solution employed a boundary vorticity method with a uniform wall heat flux and peripherally uniform wall temperature. Kalb and Seader [4] performed numerical studies for uniform wall heat flux with peripherally uniform wall temperature for Dean numbers in the range of 1-1200, Prandtl numbers of 0.005-600, and curvature ratios (r/R) of 1/10 to 1/100 for fully developed velocity and temperature fields. They found that curvature ratio parameter has negligible effect on the average Nusselt number for any given Prandtl number. They reported that local Nusselt numbers on the outer wall of the helical tube continued to increase with increasing Dean number. They also noted that the fractional increase in heat transfer coefficients is significantly greater than the fractional increase in friction losses, except for liquid metals. Laminar convective heat transfer in curved tubes was studied both experimentally and numerically by Janssen and Hoogendoorn [5] for both uniform heat flux and constant wall temperature boundary conditions. Thermal entry region was also studied. They showed that the effect of boundary conditions on laminar convection heat transfer was small. Effect of helical coil tube pitch on heat transfer and pressure drop was studied by Austen and Soliman [6] for the case of uniform wall heat flux. The results showed significant pitch effects on both of friction factor and Nusselt number at low Reynolds numbers; however these effects weakened as the Reynolds number increased. They suggested that these pitch effects are due to free convection, and the effect decreases as the forced convection becomes more dominant at higher Reynolds numbers. Liu and Masliyah [7] investigated the effect of pitch and torsion on secondary flow fields for fully developed laminar flow. Pressure drop and friction factors were also studied for fully developed laminar flow.

Later, Liu and Masliyah [8] numerically studied laminar Newtonian flow and heat transfer development using fully parabolic equations in the axial direction. Their studies also took into consideration the pitch of helix. Yamamoto et al. [9] studied transition from laminar to turbulent flow for helical coils with large curvature and large torsion. They concluded that while curvature has a stabilizing effect on the flow, the torsion had a destabilizing effect. Yamamoto et al. [10] further investigated the effect of torsion on the stability of flow by first defining a torsion parameter and then proceeded to find the critical Dean number at different torsion parameter values. They showed that as the torsion parameter increases, critical Dean number decreases at first, reaching a minimum, then began to increase again. Effect of the pitch on the Nusselt number in laminar flow of helicoidal pipes was also determined by Yang et al. [11]. Effect of the Prandtl number on heat transfer rate and on both average and local Nusselt numbers for flow in helical pipes was studied by Xin and Ebadian [12]. In their studies, different torsions and curvature ratios were considered along with three different fluids, air, water and ethylene glycol. They concluded that the peripheral Nusselt numbers for laminar flow showed larger variation for higher Prandtl and Dean numbers.

Tube in tube helical heat exchanger

Karahalios [13] studied heat transfer of a fluid flowing in a curved pipe with a solid core. The core and the curved pipe surface were at constant temperature gradient along the axial direction. Depending on Dean number, a reversal of the flow was detected in the inner portion of the bend for significantly large cores. Petrakis and Karahalios [14] studied steady annular flow of an incompressible viscous fluid in a curved pipe with a coaxial core. Their findings showed that presence of a core affects flow properties, especially at high Dean numbers. They also developed analytical expressions for axial velocity and for stream function for exponentially decaying flow in a curved annular pipe. In both works, it was shown that in some instances two additional secondary flow patterns developed resulting in a total of four vortices. Xin et al. [15] experimentally studied both single-phase and two-phase flows in helicoidally annular pipes to determine pressure drop relationships. They developed a pressure drop correlation for single-phase flow for laminar, transitional, and turbulent flow regimes. For the two-phase flow, they studied coils in both horizontal and vertical configurations and provided pressure drop correlations for each case. Petrakis and Karahalios [16] used a finite difference numerical method for flow of a viscous, incompressible fluid in a curved annular conduit with a circular cross section. The Dean range was from 96 to 8000. Various core sizes were used. Influence of annular tube contact in a helical-wound tube in tube heat exchanger was studied by Louw [17]. Comparison of such heat exchangers to aligned (concentric) devices was done experimentally to quantify the influence of annular contact on heat exchange capabilities. Nusselt numbers were used to predict convection heat transfer coefficients, and by method of comparison it was found that annular contact decreased convection from an inner tube and improved it into an annular space. Rennie [18] and Rennie and Raghavan [19] experimentally reported the heat transfer in a tube in tube heat exchanger comprised of one loop. Two heat exchanger sizes and both parallel flow and counterflow configurations were tested. They reported a little difference between the overall heat transfer coefficients for the parallel flow and counterflow configurations. However, heat transfer rates were much higher in the counterflow configuration due to larger average temperature difference between the two fluids. They also reported that increasing tube Dean numbers or annulus Dean numbers resulted in an increase in the overall heat transfer coefficient. Pressure drop and heat transfer study for tube in tube helical heat exchanger was studied by Kumar et al. [20]. The experiments were carried out in counter current mode operation with hot fluid in the tube side and cold fluid in the annulus area. Overall heat transfer coefficients were calculated and heat transfer coefficients in the inner and outer tube were determined using Wilson plots.

The above literature show that although a significant amount of research has been performed on the flow patterns and heat transfer in curved pipes and helically coiled pipes, there is still much that needs to be investigated, in particular multi tubes in tube helical heat exchangers.

2. Experimental Setup and Procedure

The experimental setup and procedure were designed to study average heat transfer coefficients and pressure drop in the annulus of multi tubes in tube liquid to liquid helical heat exchanger. The annulus fluid used was water. To maintain and control a constant heat flux boundary conditions on inner annulus surfaces, electrical heater rods are used to simulate the inner tubes of multi tubes in tube heat exchanger. To study effects of some of geometric parameters such as number of inner tubes and annulus hydraulic diameter on heat exchanger performance, different test sections with different numbers of inner tubes were designed, manufactured and tested. Namely, four multi tubes in tube heat exchangers with one, three, four and five inner tubes, denoted as coil 1, coil 2, coil 3 and coil 4, respectively, are tested. Figure 2 shows cross section views of the different arrangements of tested heat exchangers:

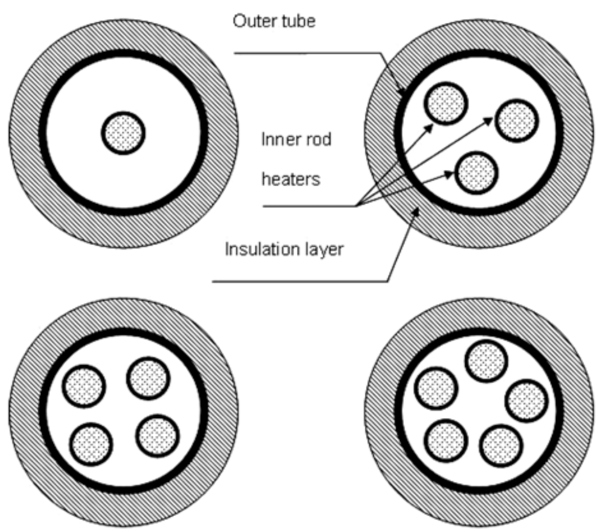

Figure 2. Section views of the tested 1, 3, 4 and 5-tubes in tube heat exchangers.

2.1. Experimental setup

A schematic diagram of the experimental setup is shown in Figure 3. The experimental setup can be divided to two main sections; cooling water circuit and test section. Cooling water circuit is an open type circuit. It consists of a constant head tank, circulating pump with a bypass line, mixing cups, test section annulus, water flow meters and a graduated vessel used for flow rate measurements. Constant head tank is a thermally insulated polyethylene water storage tank. The tank is fitted with a water level controller to assure a constant water level inside the tank during the experiment. $3/4$ hp circulating pump with a bypass line is used to control cooling water flow rate passing through test section. Two water mixing cups are inserted just upstream and downstream test section to enable measuring average water temperatures at inlet and exit of test section. Two T-type thermocouples props were inserted in the mixing cups to measure water temperatures. Cooling water flow rates were measured by collecting certain water at test section exit in a graduated 1000 ml vessel in a certain time period measured by a standard stop watch. Two acrylic variable area flow meters of different ranges are used to regulate and approach the required water flow rates. Flexible PVC tubing with standard fitting are used to connect sections of the water circuit together. Pressure drop of water flow in test section annulus was measured by an inverted U tube manometer.

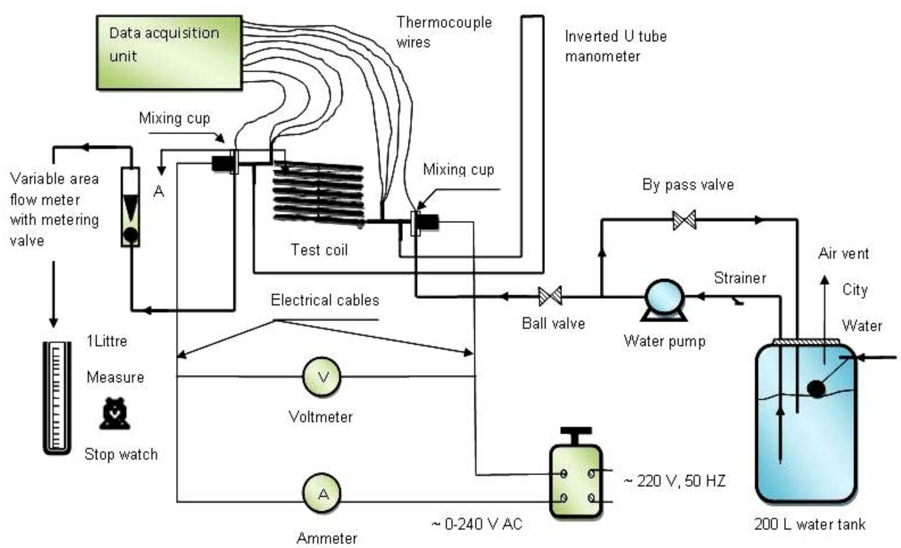

Figure 3. Experimental setup.

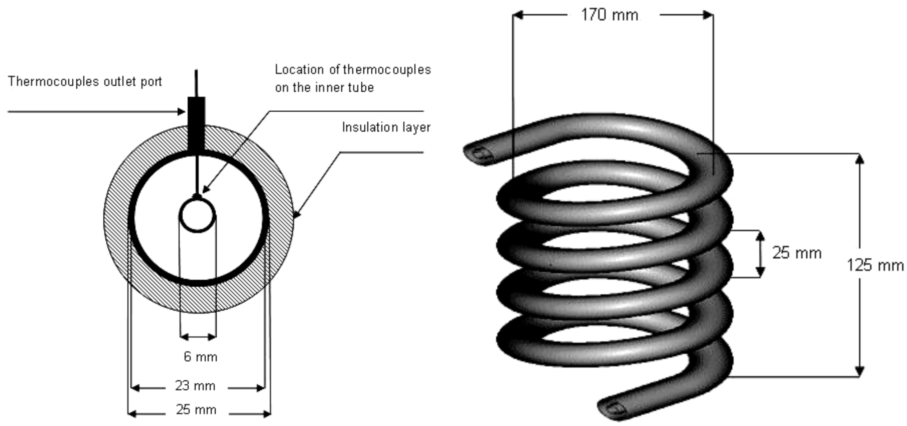

Figure 4. Physical dimensions of tube in tube helical heat exchanger.

Test section is a multi tubes in tube helical coil. The inner tube(s) was (were) simulated by a 6 mm diameter electrical heater rod(s). The heater rods were constructed from stainless steel-316 thin tubes with nickel Krum resistance wire impeded inside it and shield with a special powder. The outer tube of the helical coil was a copper tube of 23 mm and 25 mm inside and outside diameters, respectively. Four test sections of different numbers of heater rods; namely one, three, four and five were tested. The length of each heater rod and the outer tube was 3.5 meter before coiling. Heater rod(s) was (were) inserted inside the outer tube and concentric in it by guidance from both ends. The outer tube with heater rods inside it was coiled to form a helical coil of five turns and 170 mm helical coil diameter with 25 mm pitch between each turn and other. Figure 4 and Table 1 give helical coils geometrical and physical parameters, respectively. Helical coils outer surfaces were thermally insulated from all sides by 5 cm-thick glass wall thermal insulation. Heater rods were connected together in parallel to an electrical power source via a voltage transformer. The voltage transformer was used to regulate electrical power input to heater rods to achieve same required heat flux outlet from each one. Voltage drop and current carried in each electric heater rod were measured by digital ammeter and voltammeter. Two thermocouples (type T) were fixed on the surface of each heater rod at each two ends to measure the average heater rod surface temperature. The thermocouple wires were passed from the outer tube of the helical coils through holes and sleeves around the holes at the two ends of the coil as shown in Figure 4. All thermocouples were calibrated in a constant temperature path and a measurement accuracy of $\pm 0.2°C$ was obtained. All the temperature signals were acquired using a data acquisition system and sent into a PC for data recording.

Table 1. Heat exchangers physical dimensions

	D_o (mm)	D_i (mm)	do (mm)	L (mm)	D (mm)	Number of tubes	A_c (m^2)	D_h (m)	A_h (m^2)
Coil 1	25	23	6	3.5	170	1	3.87E-04	1.70E-02	6.59E-02
Coil 2	25	23	6	3.5	170	3	3.30E-04	1.03E-02	1.98E-01
Coil 3	25	23	6	3.5	170	4	3.02E-04	8.19E-03	2.64E-01
Coil 4	25	23	6	3.5	170	5	2.74E-04	6.58E-03	3.30E-01

2.2. Data reduction

Reynolds number of water flow in the test section annulus was calculated based on annulus hydraulic diameter as given by:

$$Re = \frac{\rho_w \dot{V} D_h}{A_c \mu_w}, \tag{1}$$

where ρ_w and μ_w are density and dynamic viscosity of water calculated at $(T_{wi} + T_{wo})/2$, $\dot{V}$ is water volumetric flow rate, A_c and D_h are annulus cross sectional area and annulus hydraulic diameter given by:

$$A_c = \pi(D_i^2 - nd_o^2)/4, \tag{2}$$

$$D_h = \frac{4A_c}{(\pi D_o + \pi n d_o)}, \tag{3}$$

where D_o, n and d_o are annulus outer diameter, number of inner heater rods and outer diameter of inner heater rods, respectively.

Dean number for water flow in the helical annulus is defined by equation (4). The curvature ratio (R) used in equation (4) is taken as the ratio of the annulus gap to the radius of curvature of helical coil outer tube

$$De = Re\sqrt{\frac{D_h}{2R}}. \tag{4}$$

Heat transfer rate Q was calculated by making heat balance between cooling water conditions at inlet and exit of test section and electric heat input to heater rods neglecting heat losses from test section due to well thermally insulated

$$Q = \dot{m}_w C_p (T_{wo} - T_{wi}), \tag{5}$$

$$Q = VI, \tag{6}$$

where $\dot{m}_w$ is cooling water flow rate, T_{wi}, T_{wo} are cooling water temperatures at inlet and exit of test section. C_p is specific heat of cooling water calculated at average cooling water temperature $(T_{wi} + T_{wo})/2$. V and I are voltage and current input to rod heaters, respectively. Heat transfer rate q was determined from both equations (5) and (6). The difference was usually smaller than $\pm 5\%$. A test run was repeated if the deviation was larger than $\pm 5\%$. In our calculations, the average of the two values was considered as the heat transfer rate.

The mean heat transfer coefficient was calculated from

$$\bar{h} = \frac{Q}{\pi n L d_o (T_s - T_w)}, \tag{7}$$

where T_s and T_w are average surface temperature of heater rods and mean temperature of cooling water, respectively, and L is heater rod length. T_s and T_w are calculated from

$$T_s = \frac{\sum_{j=1}^{j=n} 0.5(T_{si} + T_{so})_j}{n}, \tag{8}$$

$$T_w = (T_{wi} + T_{wo})/2, \tag{9}$$

where T_{si} and T_{so} are heater rod average surface temperature at inlet and exit of test section.

The average Nusselt number is calculated based on annulus hydraulic diameter from:

$$\overline{Nu} = \frac{\bar{h} D_h}{h_w}, \quad (10)$$

where k_w is water thermal conductivity calculated at T_w.

2.3. Experimental conditions and procedure

Experiments were carried out at an electrical power supply to heater rods in the range 527-3297 W. Reynolds number was varied in the range 350-6500. Experiment data was recorded after maintaining steady state conditions. To be sure of steady state conditions temperature readings of all thermocouples are approximately constant with time (± 0.2°C was considered for a period of time 30 minutes). After achievement of steady state conditions, the following measurements were recorded in each experiment: water flow rates, pressure drop across test section, voltage and current applied on the heater rods and all thermocouple readings. The ranges of the measured variables during all experiments were 22.2-33.3°C for T_{wi}, 24.8-62.3°C for T_{wo}, 32.6-74°C for T_s, 0.006-0.228 kg/s for $\dot{m}_w$ and 527-3297 W for electric power (*VI*). The uncertainty ranges of the measured variables during all experiments were: 0.9-0.6% for T_{wi}, 0.8-0.32% for T_{wo}, 0.61-0.27% for T_s, 0.507-0.7% for $\dot{m}_w$, 0.44-2.2% for electric power and 0.0001 m error of tube diameter measurements. Details of ranges of measured parameters and uncertainties are given in Al Shaer [21].

Using equations (2)-(10), the average Nusselt number can be put on the form $Nu = f(x_1, x_2, ..., x_n)$, where x_1 to x_n are all the variables that affect the experimental determination of *Nu*. The uncertainty ΔNu in the value of *Nu* was estimated based on the procedure of Holman and Gajda [22] and is expressed as follows:

$$\Delta N\dot{u} = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial Nu}{\partial x_i} \Delta x_i \right)^2}, \quad (11)$$

where Δx_i is the uncertainty in the x_i variable. Following the above procedure, it was found that the uncertainty for all data of $\overline{Nu}$ ranges from 5 to 7%. For detailed calculation and uncertainty analysis, kindly refer to Al Shaer [21].

3. Results and Discussions

Experimental runs were performed to study the effects of some of heat exchanger geometric parameters and fluid flow parameters; namely, number of the inner tubes, annulus hydraulic diameter, Reynolds numbers and input heat flux, on the performance of multi tubes in tube helical coil, namely on (i) heat transfer coefficient, and (ii) $\bar{h}A_h$ the factor that measures performance and compactness of heat exchangers and Nusselt number.

3.1. Variation of pressure drop

Figure 5 shows variation of cooling water pressure drop across heat exchanger against Reynolds number with coil number (number of inner tubes) as a parameter. The figure shows the increase of pressure drop with increasing both of Reynolds numbers and number of inners tubes. The results agree with the scientific facts of increasing pressure drop with the increase of the fluid velocity and the increase of the wetted perimeter. Increasing the number of inner tubes increases the wetted perimeter which in turn increases pressure drop.

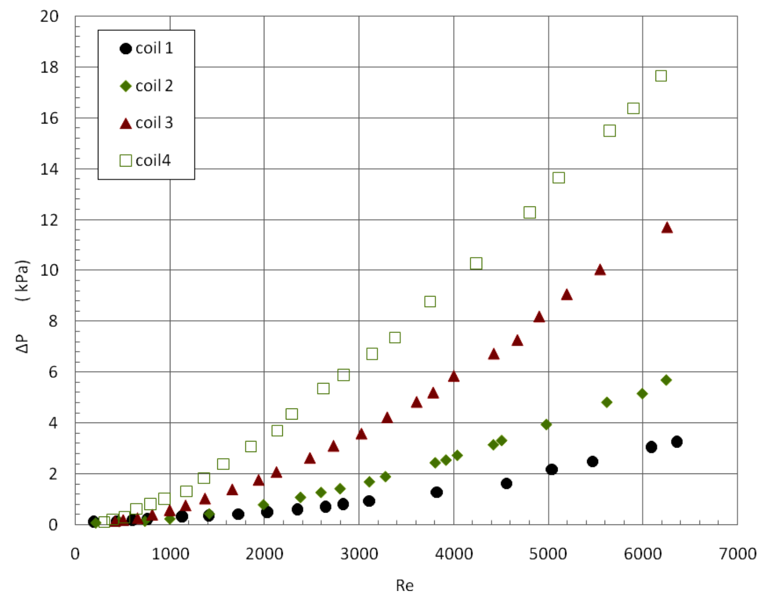

Figure 5. Variation of pressure drop ΔP with Reynolds number for different coils.

3.2. Variation of average heat transfer coefficients

Figure 6(a, b, c, d) shows variation of average heat transfer coefficients versus Reynolds number with input heat rate as a parameter for coils 1, 2, 3 and 4, respectively. The figure shows that for all coils, heat transfer coefficient increases with increasing Reynolds number and is not affected by input heat rate. Increase of heat transfer coefficient with Reynolds number agrees with scientific facts where heat transfer coefficient increases with increasing of Reynolds number due to the increase of mixing and turbulence level in the flow. Although, the studied range of Reynolds number lies in the mixed convection range for flow inside plain tubes, the present results which show the independence of heat transfer coefficient on heat flux, as shown in Figure 6, prove that the transition from mixed convection region to forced convection region for annulus flow occurs at lower Reynolds number as compared to flow inside plain tube. This can be attributed to higher mixing and turbulence level for flow in annulus helical space. According to this independence of heat transfer coefficient on heat flux, there is no need to differentiate between the results of different heat fluxes by different legends in the subsequent figures. The different heat fluxes will be presented on the figures by the same legend.

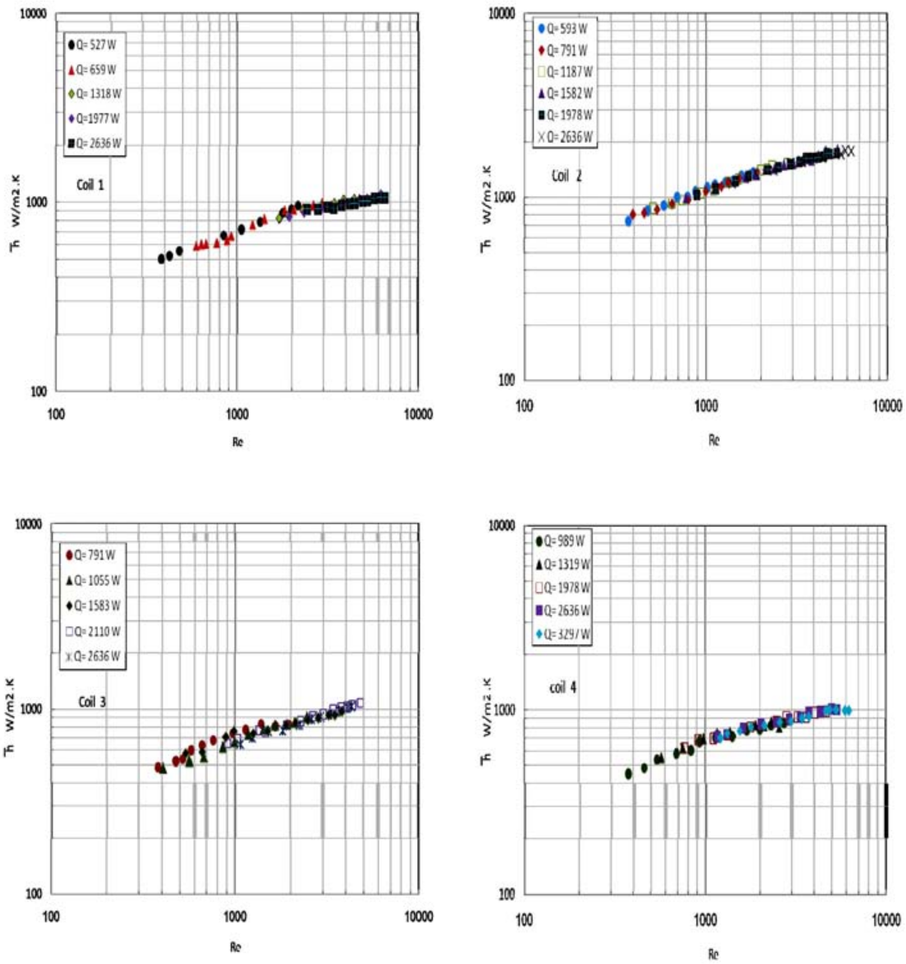

Figure 6. Variation of average heat transfer coefficients versus Reynolds for coils 1, 2, 3, and 4 at different input heat rates.

To show the effect of number of inner tubes on heat transfer coefficient, Figures 6(a) to 6(d) are re-plotted in Figure 7 with the coil number as a parameter. Figure 7 shows that heat transfer coefficient for coil 2 (number of inner tubes = 3) is significantly higher than those for other coils. Heat transfer coefficients of coils 1, 3 and 4 approach each other. The trend is the same for all Reynolds numbers. Figure 7 reveals that there is an optimum number of inner tubes of the multi tubes in tube helical heat exchanger at which heat transfer coefficient is maximum. Since number of inner tubes has a direct impact on hydraulic diameter, as defined in equation (4), an optimum number of inner tubes means an optimum hydraulic diameter at which heat transfer coefficient is maximum.

To show the effect of hydraulic diameter on heat transfer coefficient and to indicate optimal hydraulic diameter, Figure 7 is represented in Figure 8, where heat transfer coefficient is plotted versus hydraulic diameter with Reynolds number as a parameter. Figure 7 shows that optimal hydraulic diameter is independent on Reynolds number and always occurs at $D_h = 0.12\,\text{m}$ which is nearly equivalent to hydraulic diameter of coil 2. Figure 8 also shows that variation of heat transfer coefficient is parabolic with maximum value at $D_h = 0.12\,\text{m}$. The existence of optimum value for heat transfer coefficient at number of inner tubes equals three may be attributed to the optimization of the effects of flow mixing, turbulence and secondary flow on heat transfer coefficient that occurs at number of inner tubes equals three.

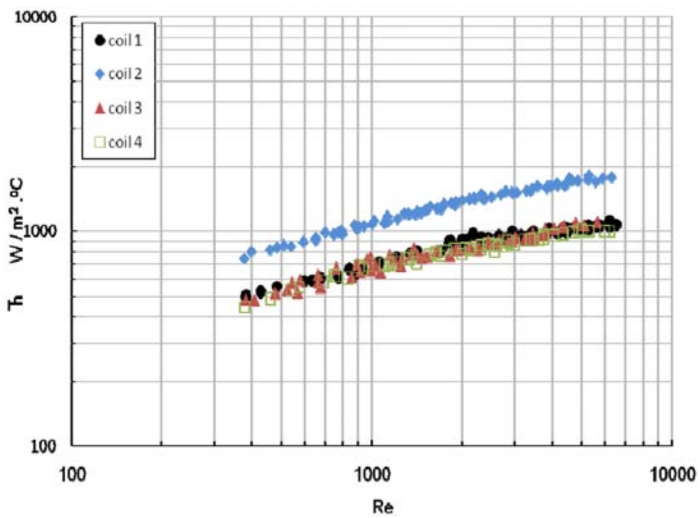

Figure 7. Variation of average heat transfer coefficients versus Reynolds with number of inner tubes as a parameter.

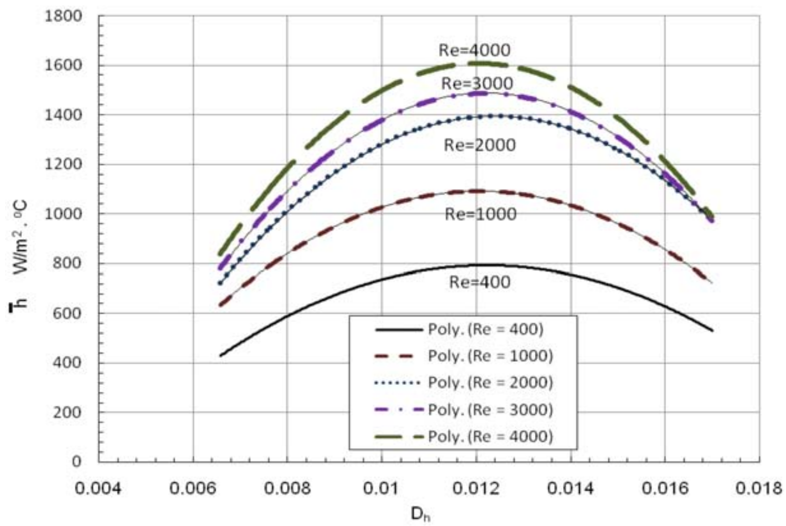

Figure 8. Variation of average heat transfer coefficients versus hydraulic diameter at different Reynolds number.

3.3. Variation of $\bar{h}A_h$

In addition to heat transfer coefficient, $\bar{h}A_h$ can be used to measure performance and compactness of heat exchangers. For same volume occupied by heat exchangers, increasing $\bar{h}A_h$ means the increase of the capability of the heat exchanger to heat transfer. In the current study, the volume of the heat exchanger is fixed for all coils and equals to the volume occupied by the outer helical shell. Therefore coil which gives higher $\bar{h}A_h$ will be recommended. Figure 9 shows variation of $\bar{h}A_h$ versus Reynolds number with coil number as a parameter. Figure 9 shows that coils 2 and 4 which have number of inner tubes equals 3 and 5, respectively, have a higher $\bar{h}A_h$ as compared to coils 1 and 3 which have number of inner tubes equals 1 and 4, respectively. The trend is the same at all Reynolds numbers. Figure 9 also shows that $\bar{h}A_h$ are approximately the same for coils 2 and 4. Therefore, from heat transfer point of view in compact heat exchangers, coils number 2 and 4 have the same and best performance as compared to other coils. However, coil 2 is superior since it has smaller number of inner tubes (three inner tubes) as compared to coil 4 (five tubes). This means that cost and cooling water pressure drop of coil 2 is smaller than that of coil 4. The above discussion shows that for the studied multi tubes in tube helical heat exchanger, coil 2 which has three inner tubes has the best optimal performance; maximum heat transfer, maximum $\bar{h}A_h$, minimum pressure drop and minimum cost.

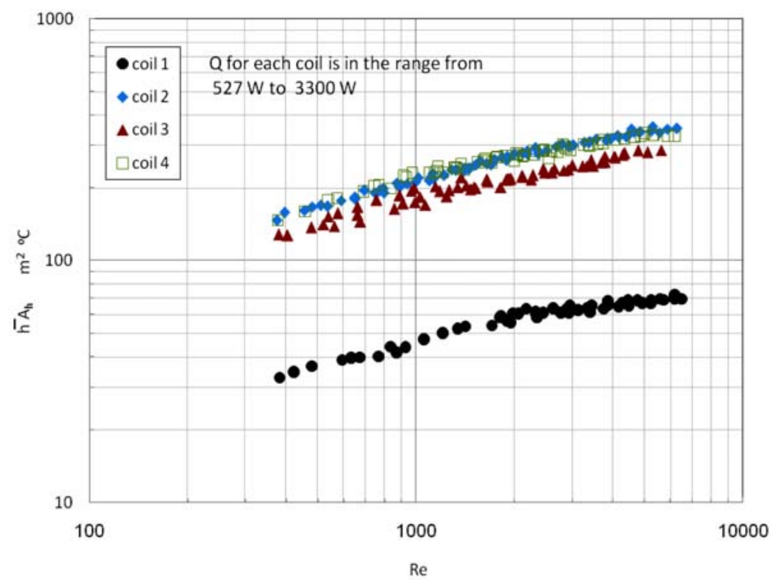

Figure 9. Variation of $\bar{h}A_h$ versus Reynolds number with coil number as a parameter.

3.4. Variation of average Nusselt number

Average Nusselt number, defined by equation (10), is mainly function of heat transfer coefficient and annulus hydraulic diameter. Annulus hydraulic diameter is not the same for all coils but it varies according to number of inner tubes as shown in equation (3) that reveals the decrease of hydraulic diameter with increasing number of inner tubes. Figure 10 shows variation of average Nusselt number versus Reynolds number with coil number (number of inner tubes) as a parameter. The figure shows that for all coils, Nusselt number increases with increasing Reynolds number. The figure also shows that Nusselt numbers of coils 1 and 2, which are approximately have the same values, are higher than those of coils number 3 and 4. The figure also shows that coil 3 has higher Nusselt number than that of coil 4. This effect of coil number on Nusselt number can be attributed to the effect of coil number on heat transfer coefficient as previously discussed and the decrease of hydraulic diameter with the increase of number of inner tubes.

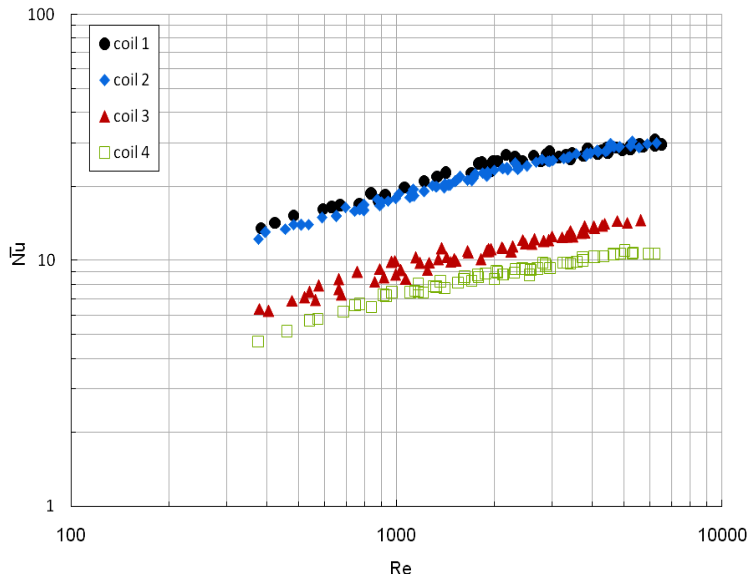

Figure 10. Variation of average Nusselt number with Reynolds number with coil number as a parameter.

The experiments were conducted at various input heat fluxes and various water cooling flow rates. Varying both of heat flux and water cooling flow rate varies heat transfer surface temperature and mean fluid temperature. These variations will lead to a variation in the average liquid temperature $(T_{wi} + T_{wo})/2$ at which the properties are calculated. This means that during the experiments program Prandtl number cannot be kept constant. It varies according to the variation of input heat flux and cooling water flow rate. Figure 11 shows variation of average Nusselt number versus Prandtl number with coil number as a parameter at Reynolds numbers of 2000 and 3000, respectively, as examples. The figure shows slightly increase of Nusselt number with Prandtl number. Trend is nearly the same for all coils and at all

Reynolds numbers. The trend agrees with the scientific fact and the available experimental work for forced convection in tubes which showed that Nusselt number varies with Prandtl number according to the power low $(\overline{Nu} \propto Pr^{n})$.

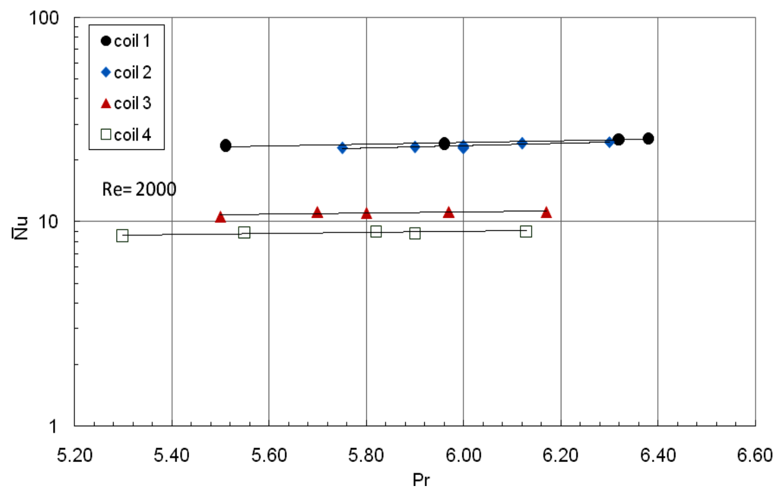

Figure 11. Variation of $\overline{Nu}$ versus Pr for all coils at $Re = 2000$.

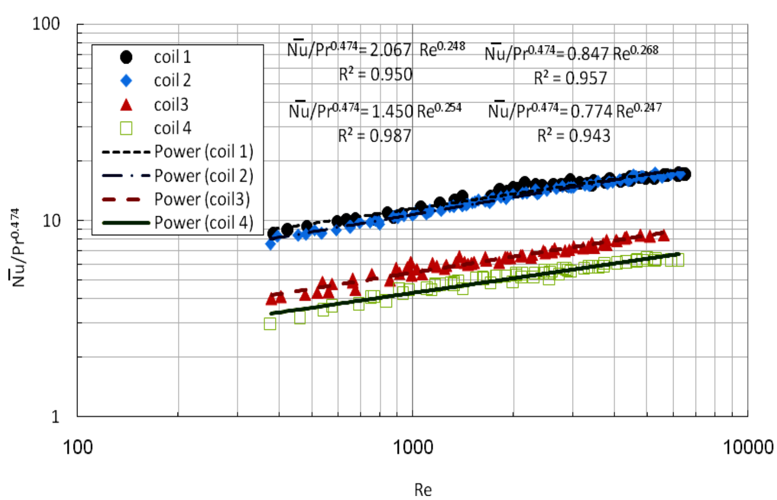

Figure 12. Variation of $(\overline{Nu}/Pr^{0.474})$ with Reynolds number for each coil.

Making a regression analysis for our present experimental results for all coils and at all Reynolds numbers, the power exponent (n) of the variation of Nusselt number with Prandtl number was found equals to 0.474. To eliminate effect of variation of Prandtl number on effects of Reynolds number and heat flux on Nusselt number, $\overline{Nu}/Pr^{0.474}$ is plotted versus *Re* at all heat fluxes for all coils in Figure 12. Figure 12 also shows regression lines for each coil data. The regression shows that for all coils, $\overline{Nu}/Pr^{0.474}$ increases with increasing *Re* according to a power low as given by the following equation:

$$\overline{Nu}/Pr^{0.474} = C\,Re^{m}. \tag{12}$$

Table 2 gives the coefficients *C* and *m* for the different coils. As shown in Table 2, the exponent *m* is approximately constant and does not depend on the number of inner tubes. This indicates that trend of the variation (slop of the line) of $\overline{Nu}$ with *Re* does not depend on number of inner tubes of the coil. The average values of the exponent *m* for all coils can be considered as 0.251. This value will be taken as an exponent of the power variation of $\overline{Nu}$ with *Re* for all coils. Also, Table 2 shows that the value of the constant *C* strongly depends on number of inner tubes of the coil. This means that Nusselt number strongly depends on annulus hydraulic diameter of the multi tubes in tube helical coil.

Table 2. Values of *C* and *m* of equation (12) for all coils

Coil number	Number of tubes	D_h (m)	C	m	R^2
1	1	1.70E-02	2.067	0.248	0.950
2	3	1.03E-02	0.847	0.268	0.957
3	4	8.19E-03	1.450	0.254	0.987
4	5	6.58E-03	0.774	0.247	0.943

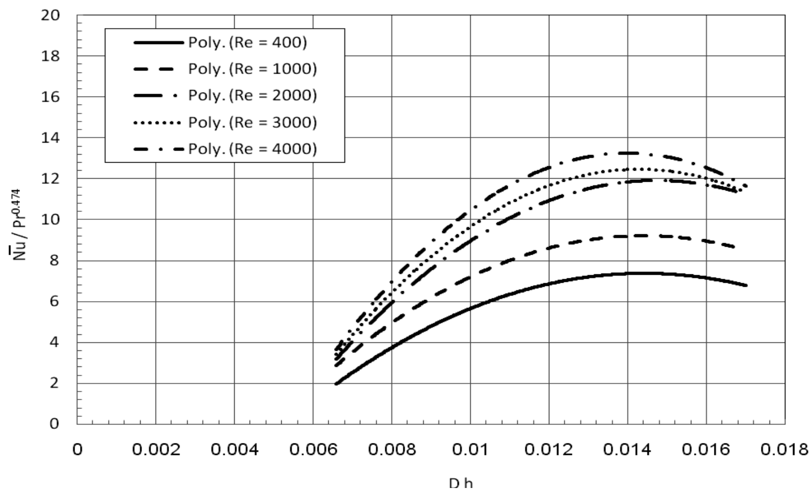

Figure 13. Variation of $(\overline{Nu}/Pr^{0.474})$ with hydraulic diameter at various Reynolds numbers.

Equation (12) and Table 2 are used to predict effect of annulus hydraulic diameters of multi tubes in tube helical coil on Nusselt number. Figure 13 shows variation of $\overline{Nu}/Pr^{0.474}$ versus hydraulic diameter for various Reynolds numbers. The figure indicates that there is a hydraulic diameter at which $\overline{Nu}/Pr^{0.474}$ is maximum. This hydraulic diameter can be considered as coil optimal hydraulic diameter. The figure shows that this optimum hydraulic diameter always occurs at $D_h = 0.0142\,\mathrm{m}$ independent on Reynolds number. To correlate variation of Nusselt number with hydraulic diameter, $\overline{Nu}/(Pr^{0.474}Re^{0.251})$ is plotted versus D_h for all experimental data in Figure 14. Figure 14 shows that variation of $\overline{Nu}/(Pr^{0.474}Re^{0.251})$ with hydraulic diameters is parabolic and has its maximum value at $D_h = 0.0142\,\mathrm{m}$ with regression correlation:

$$\overline{Nu} = (-20555D_h^2 + 583.6D_h - 2.486)Re^{0.251}Pr^{0.474}. \tag{13}$$

The above equation is deduced from data of ranges $376 < Re < 6516$ and $5.3 < Pr < 6.37$. R^2 value of this regression is 0.908 which means that scatter of experimental data around regression line is high. Careful examination of Figure 14 reveals that data of coil 3 (number of inner tubes = 4) are data with high scatter and the scatters of other coils data are small enough. Excluding data of coil 3, the only coil with even number of inner tubes, and make curve fitting of other coils data as shown in Figure 15, R^2 value will become 0.989. This means that trend of variation of $\overline{Nu}/(Pr^{0.474}Re^{0.251})$ with D_h for multi tubes in tube helical coil with odd number of inner tubes differs than that of even number of inner tubes. This can be attributed to flow pattern and flow regime in coil annulus which strongly depend on distribution of inner tubes inside helical outer tube, where inner tubes distributions in case of odd number always assure presence of a tube inside the core of the helical shell. Figure 14 shows that for multi tubes in tube helical coils with odd number of inner tubes, the maximum Nusselt number always occurs at $D_h = 0.0138\,\mathrm{m}$. Data of the Nusselt number for multi tubes in tube helical coil with odd number of inner tubes can be correlated by the following equation as shown in Figure 15:

$$\overline{Nu} = (-22308D_h^2 + 614.6D_h - 2.493)Re^{0.248}Pr^{0.474}. \tag{14}$$

The above equation is deduced from data of ranges $376 < Re < 6516$ and $5.3 < Pr < 6.37$. Comparison of predictions of equations (13) and (14) with experimental data are shown in Figures 16-19, respectively. Figures 16 and 17 show that equation (13) can predict Nusselt number for all coils within $+/-29.5\%$ Figures 18 and 19 show that equation (14) can predict Nusselt number for coils with odd number of inner tubes within $+/-8.5\%$.

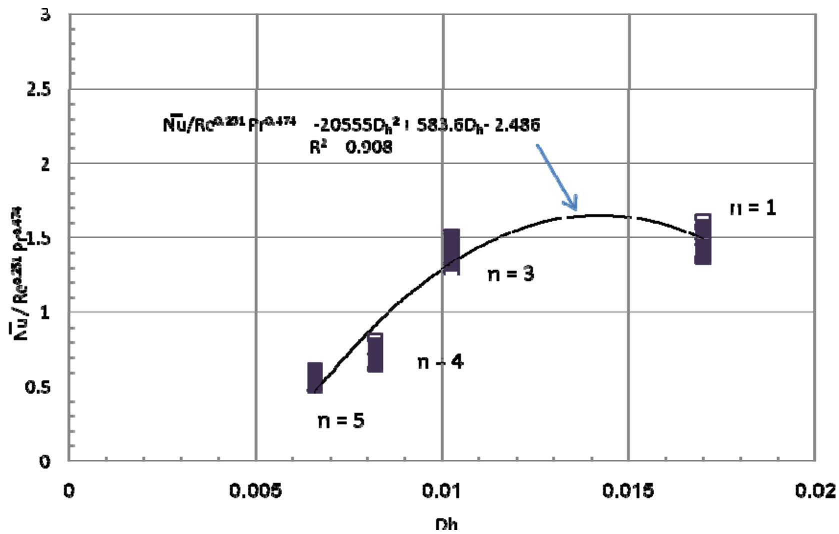

Figure 14. Variation of $\overline{Nu}/(Pr^{0.474}Re^{0.251})$ with annulus hydraulic diameter.

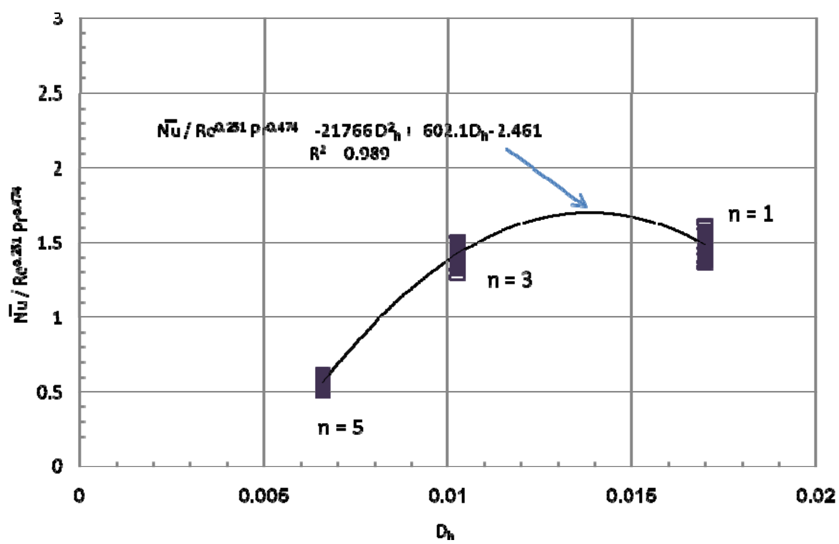

Figure 15. Variation of $\overline{Nu}/(Pr^{0.474}Re^{0.251})$ with annulus hydraulic diameter of coils with odd number of inner tubes.

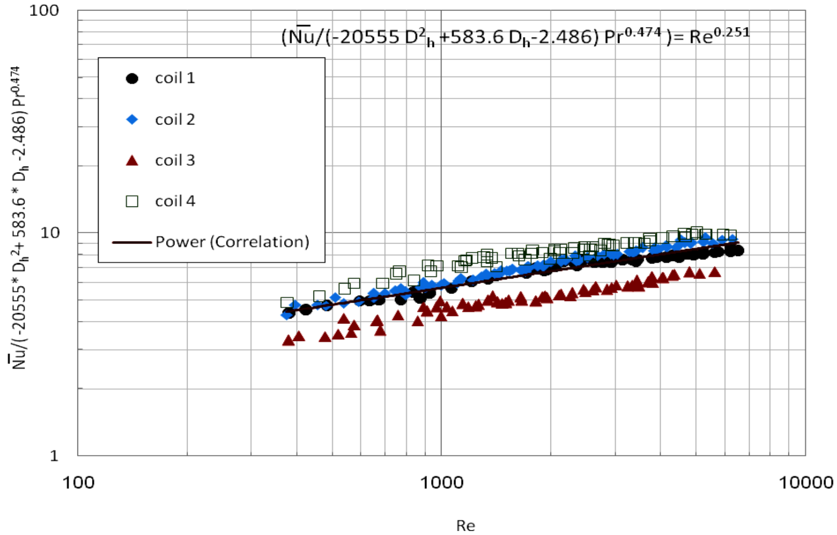

Figure 16. Correlation of Nusselt number with Reynolds number for coils with even or odd number of inner tubes.

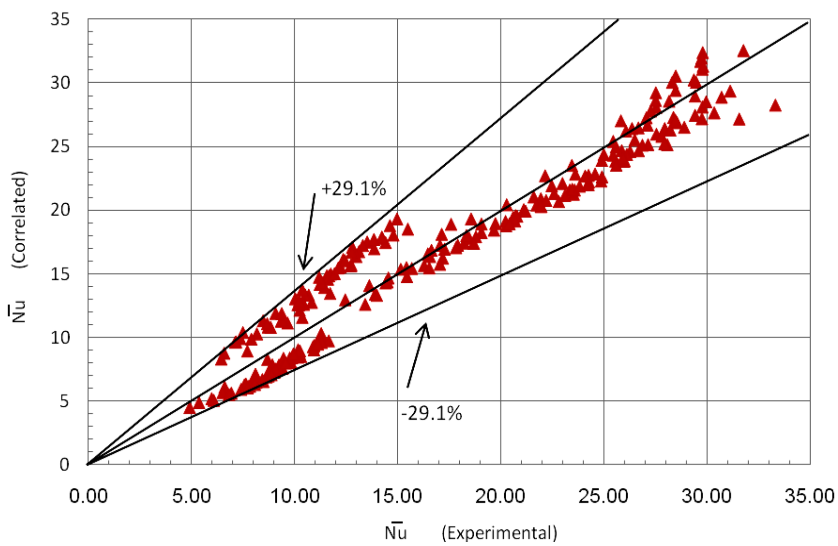

Figure 17. Deviation between experimental results and prediction of equation (13).

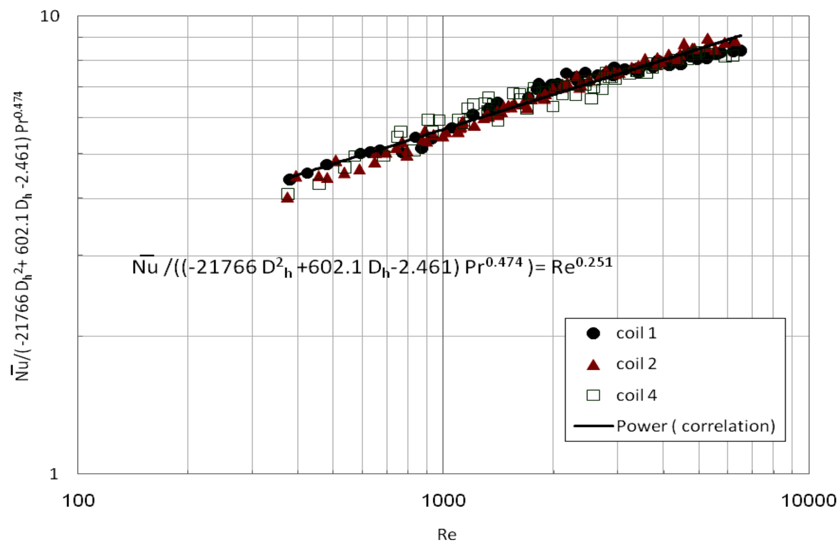

Figure 18. Correlation of Nusselt number with Reynolds number for coils with odd numbers of inner tubes.

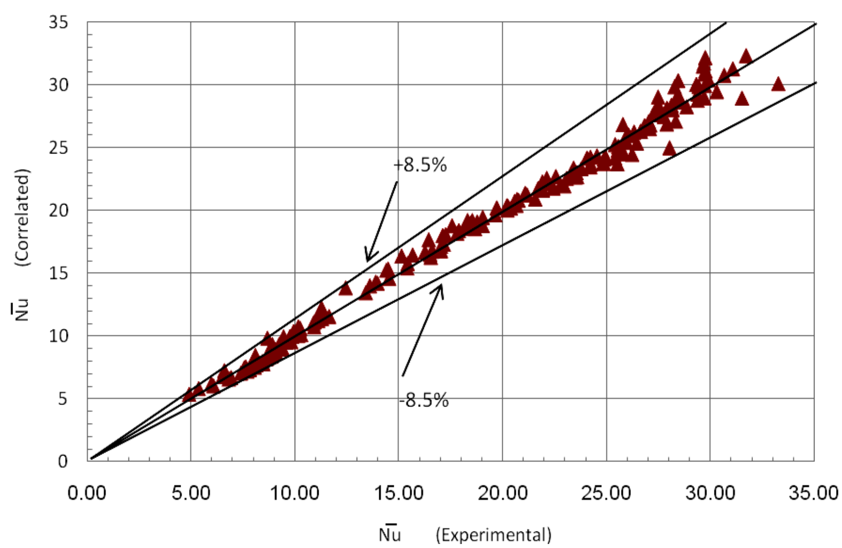

Figure 19. Deviation between experimental results and prediction of equation (14).

4. Summary and Conclusions

Experimental investigation of heat transfer and pressure drop characteristics of a multi tubes in tube helically coiled heat exchanger has been conducted to investigate effects of heat exchanger geometric parameters and fluid flow parameters on the performance of the heat exchanger. Different coils with different numbers of inner tubes, namely 1, 3, 4 and 5 tubes, were tested. The results showed that (a) the coil that had three inner tubes has higher heat transfer coefficient and bitter heat transfer characteristics as compared with other coils, (b) coils which have number of inner tubes equals 3 and 4, respectively, have higher values of $\bar{h}A_h$ (the parameter used to measure the performance and compactness of compact heat exchanger) as compared to other coils, (c) for all coils and at the tested ranges of Reynolds number and heat flux, the heat flux has no effect on the heat transfer coefficients, and (d) the pressure drop increases with increasing both of Reynolds number and number of inner tubes. Correlations of average Nusselt number were deduced from experimental data in terms of Reynolds number, Prandtl number and coil hydraulic diameter. The correlation prediction was compared with experimental data and the comparison was fair enough.

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